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Full Length Research Paper

Auxetic behavior of a thermoelastic layered plate

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The aim of this paper is to study the auxetic and non-auxetic behavior of a layered plate in context of coupled thermoelasticity. For this problem, temperature variation and thermal stresses are analyzed in a state of plane stress. Results are obtained for a layered plate subjected to a sudden uniform normal pressure and for obtained results graphs are depicted.

Key words: Auxetic behavior, uniform normal pressure, Poisson's ratio, plane stress.

INTRODUCTION

Modern technology requires new materials of special properties. One of the reasons for interest in materials of unusual mechanical properties comes from the fact that they can be used as matrices to form composites with other materials of other required properties - electric, magnetic, etc. A new field of research is to study materials exhibiting negative Poisson's ratio, named auxetics by Lakes (1987a and 1987b), which, in contrast to conventional materials (like rubber, glass, metals, etc.), expand transversely when pulled longitudinally and contract transversely when pushed longitudinally. These types of materials get fatter when they are stretched, or become thinner when compressed. Auxetic materials have been known for over 100 years by Love, but were not given much attention. This type of material can be found in some rock and minerals, even animals such as the skin covering of a cow's teats. To date, a wide variety of auxetic materials has been fabricated and studied, including polymeric and metallic foams, microporous polymers, carbon fibre laminates and honeycomb structures by Evan et al. (1991).

Over the past three decades, developments in structural engineering design and technology mainly in the aircraft, automobile, sports and leisure equipment industries have demanded the development of new, high performance materials to meet higher engineering specifications. The general requirements of such materials have been to provide a combination of high stiffness and strength with significant weight savings, resistance to corrosion, chemical resistance, low maintenance and improved reliability which, overall, would reduce maintenance costs. Evan (1990) investigated that the auxetic materials are of interest due to the possibility of enhanced mechanical properties such as shear modulus, plane strain fracture toughness and indentation resistance. Therefore, studying these non-conventional materials is indeed important from the point of view of fundamental research and possibly practical applications, particularly in medical, aerospace and defense industries. In fact, some materials with negative Poisson's ratio have been used in applications such as pyrolytic graphite for thermal protection in aerospace, large single crystals of Ni_3Al in vanes for aircraft gas turbine engines. A variety of auxetic materials and structures have been discovered, manufactured or synthesized within the last twenty years, ranging from the molecular level, up through the microscopic and right up to genuinely macroscopic level. Applications of materials with negative Poisson's ratios may be categorized based on:

(i) The Poisson's ratio,
(ii) The superior toughness, resilience, and tear resistance which have been observed in these materials, and
(iii) The acoustic properties associated with the vibration of ribs in the material.

The Poisson's ratio influences deformation kinematics in ways which may be useful, and it influences the distribution of stresses. For example, stress concentration factors are reduced in some situations, but increased in others, when Poisson's ratio is negative. Materials with negative Poisson's ratios can quantitatively improve the performance of applications.

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LITERATURE REVIEW

The negative Poisson's ratio polyurethane (PU) foam with re-entrant structures, a review dealing with auxetic materials, their fabrication methods, implications of these unusual physical properties are studied by Lakes (1993a) and (1993b). A theoretical and experimental investigation of a two-dimensional chiral honeycomb and materials with negative Poisson ratios have high shear rigidities – a useful property for many types of structural and functional materials is shown theoretically by Prall et al. (1997) and Gaoyuan et al.(1998). A novel model is proposed to explain previously fabricated foams with a negative Poisson's ratio and the strain-dependent Poisson's function behavior of honeycomb and foam materials is described by Smith et al. (2000). Alderson et al. (2001) studied the filter performance of auxetic materials from the macro-scale to the nano-scale. The existence of the counter intuitive property for which a material laterally shrinks when compressed and expands when stretched for crystalline solids of the monoclinic symmetry class, is presented by Rovati (2004). Materials with negative Poisson's ratio have been modeled largely by regular 2D networks and the results for extreme Poisson's ratios and related elastic crystal are presented by Gaspar et al. (2005) and Guo et al. (2006) respectively. The compressive mechanical properties of uniform and density-graded Al-6061 foams are investigated by Brothers et al. (2008) in which the uniform foam shows compressive behavioral characteristics of conventional (uniform) metallic foams with a near constant plateau stress, the density-graded foam exhibits a smoothly rising plateau stress. Strek et al. (2008) studied the mechanical behavior of a thick elastic plate, made of isotropic material and a layered composite with alternating layers of materials with negative and positive Poisson's ratio is presented by Gatt et al. (2009). The behavior of bi-material strips when subjected to changes in external hydrostatic pressure and a problem of a multilayered plate is investigated by Kocer et al. (2009) and Bhullar et al. (2009) respectively.

In the present paper auxetic behavior of a layered plate subjected to a sudden uniform normal pressure, thermal stresses and temperature variation for auxetic and non-auxetic materials and their comparison is studied.

MODEL AND METHODS

The constitutive equations for a homogeneous linear isotropic elastic solid given by Nowinski (1978) are

$$\theta_{,ii} = \frac{1}{\kappa}\frac{\partial\theta}{\partial t} - \frac{\rho}{k}h + \frac{E\alpha T_o}{(1-2\nu)k}\frac{\partial e_{ii}}{\partial t} \quad (1)$$

$$u_{i,jj} = \frac{1}{1-2\nu}u_{j,jj} + \frac{\rho}{\mu}b_i - \frac{2(1+\nu)\alpha}{(1-2\nu)k}\theta_i = \frac{\partial^2 u_i}{\partial t^2} \quad (2)$$

$$\tau_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} + \beta\theta\delta_{ij} \quad (3)$$

Assuming that both the displacement and the temperature fields are dependent on the field variable x and time t only, the displacement field has component only in x-direction. The geometry of the problem is shown in Figure 1.

In the following discussion, the suffix j indicates the values of field functions and material constants associated with j^{th} layer. In the j^{th} layer, in the absence of body forces and internal heat sources, the field Equations (1) and (2) in non-dimensional form (Appendix I) are:

$$\frac{\partial^2\theta_j}{\partial x^2} - \frac{1}{\kappa}\frac{\partial\theta_j}{\partial t} - \delta_1\frac{\partial^2 u_j}{\partial x\partial t} = 0 \quad (4)$$

$$\frac{\partial^2 u_j}{\partial x^2} - \delta_2\frac{1}{\kappa}\frac{\partial\theta_j}{\partial t} - \delta_3\frac{\partial^2 u_j}{\partial t^2} = 0 \quad (5)$$

where

$$\delta_1 = \frac{E_j\alpha_j T_o}{2(1+\nu_j)k_j}, \qquad \delta_2 = \frac{\alpha_j(1+\nu_j)}{(1-\nu_j)} \quad \text{and}$$

$$\delta_3 = \left(\frac{E_j(1-\nu_j)}{\rho(1+\nu_j)(1-2\nu_j)}\right)^{\frac{1}{2}}$$

The normal stress along the $x-axis$ is

$$\tau_{xx} = \frac{E_j(1-\nu_j)}{\rho(1+\nu_j)(1-2\nu_j)}\frac{\partial u_j}{dx} - \frac{E_j\alpha_j}{(1-2\nu_j)}\theta_j \quad (6)$$

and also from Equation (3), in the absence of displacements parallel to the plane $x=0$, we have

$$\tau_{yy} = \tau_{zz} = \lambda\frac{\partial u}{dx} - \frac{E}{(1-2\nu)}\theta \quad (7)$$

We use the Laplace transform pair

$$\bar{f}(x,s) = \int_0^\infty f(x,t)e^{-st}dt,$$

$$f(x,t) = \frac{1}{2\pi\iota}\int_{\gamma-\iota\infty}^{\gamma+\iota\infty}\bar{f}(x,s)e^{st}ds$$

to solve the problem. Applications of Laplace transform to the Equations (4) and (5) yields

$$\frac{\partial^2\bar{\theta}_j}{\partial x^2} - \frac{s}{\kappa}\bar{\theta}_j - \delta_1 s\frac{\partial\bar{u}_j}{\partial x} = 0 \quad (8)$$

$$\frac{\partial^2\bar{u}_j}{\partial x^2} - \delta_2\frac{\partial\bar{\theta}_j}{\partial x} - \delta_3 s^2\bar{u}_j = 0 \quad (9)$$

Upon solving simultaneous equations (8) and (9), we get the following equations in $\bar{\theta}_j$, $\bar{u}_j$

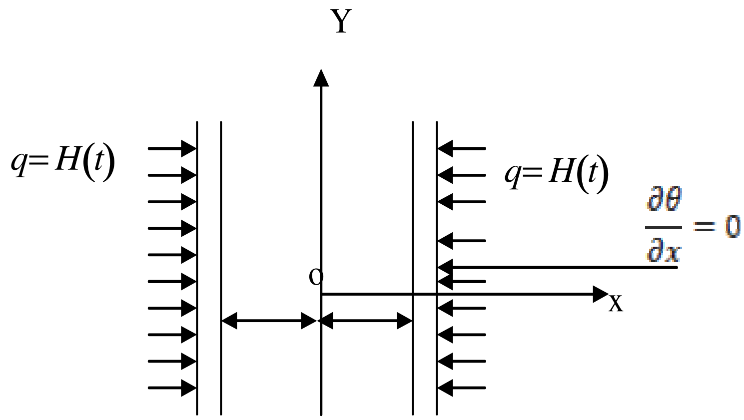

Figure 1. A layered plate subjected to pressure.

$$\frac{\partial^4\bar{\theta}_j}{\partial x^4} - s\left(\frac{1}{\kappa} + \delta_1\delta_2 + \delta_3\, s\right)\frac{\partial^2\bar{\theta}_j}{\partial x^2} + \frac{s^3\delta_3}{\kappa}\bar{\theta}_j = 0 \tag{10}$$

$$\frac{\partial^4\bar{u}_j}{\partial x^4} - s\left(\frac{1}{\kappa} + \delta_1\delta_2 + \delta_3\, s\right)\frac{\partial^2\bar{u}_j}{\partial x^2} + \frac{s^3\delta_3}{\kappa}\bar{u}_j = 0 \tag{11}$$

Here, Equations (10) and (11), are in same form. Assuming the usual exponential representations

$\bar{\theta}_j, \bar{u}_j = e^{\beta_j x}$, provides the characteristic equation for the exponent, β_j^2

$$\beta_j^4 - \frac{s}{\kappa}(1 + \varepsilon + \delta_3\, s\kappa)\beta_j^2 + \frac{s^3\delta_3}{\kappa} = 0 \tag{12}$$

where, ε is coupling coefficient. Equation (12) is quadratic in β_j^2 whose sum and product of roots is

$$\beta_{j1}^2 + \beta_{j2}^2 = \frac{s}{\kappa}(1 + \varepsilon + \delta_3\, s\kappa), \qquad \beta_{j1}^2\beta_{j2}^2 = \frac{s^3\delta_3}{\kappa} \tag{13}$$

Evidently, the roots of bi-quadratic equation (13) and general solution for equations (10) and (11) are given in equations (14) and (16) respectively:

$$\beta_{j1,2} = \sqrt{\frac{s}{2\kappa}(1 + \varepsilon + \delta_3\, s\kappa) \pm\ (1 + \varepsilon + \delta_3\, s\kappa)^2 - +\delta_3 s\kappa} \tag{14}$$

$$\bar{\theta}_j = A_{1j}\operatorname{csch}\beta_{j1}x + A_{2j}\sinh\beta_{j1}x + A_{3j}\operatorname{csch}\beta_{j2}x + A_{4j}\sinh\beta_{j2}x \tag{15}$$

$$\bar{u}_j = B_{1j}\operatorname{csch}\beta_{j1}x + B_{2j}\sinh\beta_{j1}x + B_{3j}\operatorname{csch}\beta_{j2}x + B_{4j}\sinh\beta_{j2}x$$

(16)Assuming that at time, $t = 0$, a layer of an elastic material occupying the region $-\frac{h}{2} \le x \le -\frac{h}{2}$ is subjected to a sudden uniform normal pressure of intensity q applied at the faces $x = \pm\frac{h}{2}$. The faces are thermally insulated and the temperature of the layer is $\theta = 0$, prior to the application of the load as shown in Figure 1, and the appropriate boundary conditions are:

$$\tau_{xx}\left(\pm\frac{h}{2}, t\right) = -qH(t)\ ,\ -\frac{\partial\theta(x,t)}{\partial x} = 0 \quad \text{for} \quad x = \pm\frac{h}{2} \tag{17}$$

In the case of a layer, Equation (16) and (17) will take the form as:

$$\bar{\theta}_j = A_{11}\operatorname{csch}\beta_{11}x + A_{21}\sinh\beta_{11}x + A_{31}\operatorname{csch}\beta_{12}x + A_{41}\sinh\beta_{12}x \tag{18}$$

$$\bar{u}_j = B_{11}\operatorname{csch}\beta_{11}x + B_{21}\sinh\beta_{11}x + B_{31}\operatorname{csch}\beta_{12}x + B_{41}\sinh\beta_{12}x \tag{19}$$

The integration constants $A_{11}, A_{31}, B_{11}, B_{31},$ in Equation (18) and (19) vanish due to symmetry of the problem with respect to the plane $x = 0$, therefore, the solution takes the simple form:

$$\bar{\theta}_1 = A_{11}\cosh\beta_{11}x + A_{31}\cosh\beta_{12}x \tag{20}$$

$$\bar{u}_1 = B_{21}\sinh\beta_{11}x + B_{41}\sinh\beta_{12}x \tag{21}$$

Upon substituting the Equation (21) in second boundary condition given in Equation (17) we get:

$$\bar{\theta}_1 = A_{11}\left(\cosh\beta_{11}x - \frac{\beta_{11}\sinh\frac{\beta_{11}h}{2}}{\beta_{21}\sinh\frac{\beta_{12}h}{2}}\cosh\beta_{12}x\right) \tag{22}$$

Furthermore, for Equations (22), (8) and (9) the values of integration constants are

$$B_{21} = A_{11}\frac{\beta_{11}^2 - \frac{s}{\kappa}}{s\delta_1\beta_{11}} \tag{23}$$

Table 1. Numerical data for studying auxetic and non-auxetic material.

Material constant	Alumina/Silicon
E, Young's modulus	71.7×10^{9} [Pa]
q, pressure	10^{6} [Pa]
Linear thermal expansion α	7.2×10^{-6} /k°
Mass density ρ	3.88×10^{3} kg/m^{3}
Specific heat at constant volume C_e	0.2×10^{3} J/kg °C
Thermal conductivity k	30.3 W/mk°

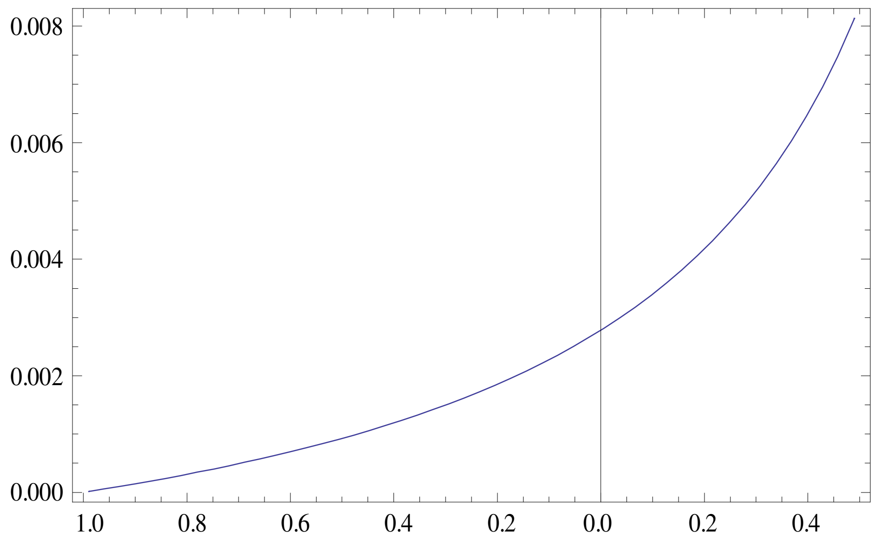

Figure 2. Comparison of temperature variation in auxetic and non-auxetic material.

$$B_{41} = A_{11}\frac{\beta_{11}}{s\delta_1\beta_{12}}\frac{\sinh\left(\frac{\beta_{11}h}{2}\right)}{\sinh\left(\frac{\beta_{12}h}{2}\right)}\left(\frac{s}{k\beta_{12}} - \beta_{12}\right) \quad (24)$$

Application of Laplace transform to the first boundary condition in Equation (18) gives

$$\bar{\tau}_{xx}\left(\pm\frac{h}{2}, s\right) = -\frac{q}{s} \quad (25)$$

Upon using Equation (25) and pair of Laplace functions the value of A_{11} 's is:

$$A_{11} = -\frac{qE(1-2\nu)}{E\alpha\Delta}\beta_{12}\sinh\left(\frac{\beta_{12}h}{2}\right) \quad (26)$$

where,

$$\Delta = \left(\kappa\beta_{11}^2 - s - \varepsilon s\right)\beta_{12}\cosh\left(\frac{\beta_{11}h}{2}\right)\sinh\left(\frac{\beta_{12}h}{2}\right) - \left(\kappa\beta_{11}^2 - s - \varepsilon s\right)\beta_{11}\cosh\left(\frac{\beta_{12}h}{2}\right)\sinh\left(\frac{\beta_{11}h}{2}\right) \quad (27)$$

Upon solving Equations (1) - (3) we obtained following expressions:

$$\bar{\theta}(x,s) = \frac{q\varepsilon(1-2\nu)}{E\alpha}\left[\frac{\beta_{11}\sinh\left(\frac{\beta_{11}h}{2}\right)\cosh\beta_{12}x - \beta_{12}\sinh\left(\frac{\beta_{12}h}{2}\right)\cosh\beta_{11}x}{\Delta}\right] \quad (28)$$

where, $\varepsilon = \kappa\delta_1\delta_2$ stands for the coupling parameter. To obtain the results for temperature field for large values of time implies that the parameter *s* tends to zero [Appendix I(b)] . After some tedious calculations, corresponding to the stationary state setting in after an infinitely long time, Equation (28) can be written in the following form:

$$\theta(x,\infty) = q\frac{\varepsilon(1-2\nu)}{E\alpha(1+\varepsilon)} \quad (29)$$

At time $t = \infty$ the action of pressure q becomes stationary and the entire layer subjected to the uniform compression $\tau_{xx} = -q$. Also at time $t = \infty$, from Equation (5) we obtain

$$\frac{\partial u}{\partial x} = \frac{(1+\nu)}{(1-\nu)}\left(\alpha\theta_\infty - q\frac{(1-2\nu)}{E}\right) \quad (30)$$

Further at time $t = \infty$, Equation (6) together with Equation (30) we have the following expression

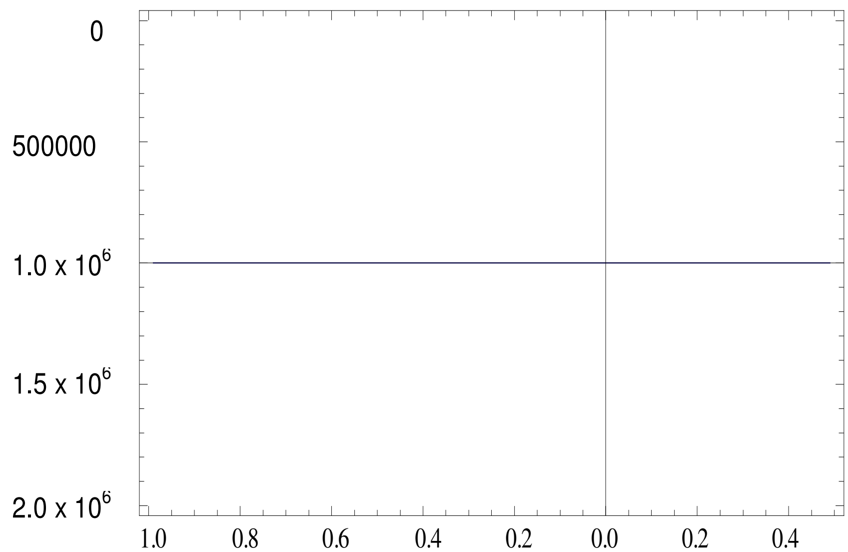

Figure 3. Comparison of stress component τ_{xx} in auxetic and non-auxetic material.

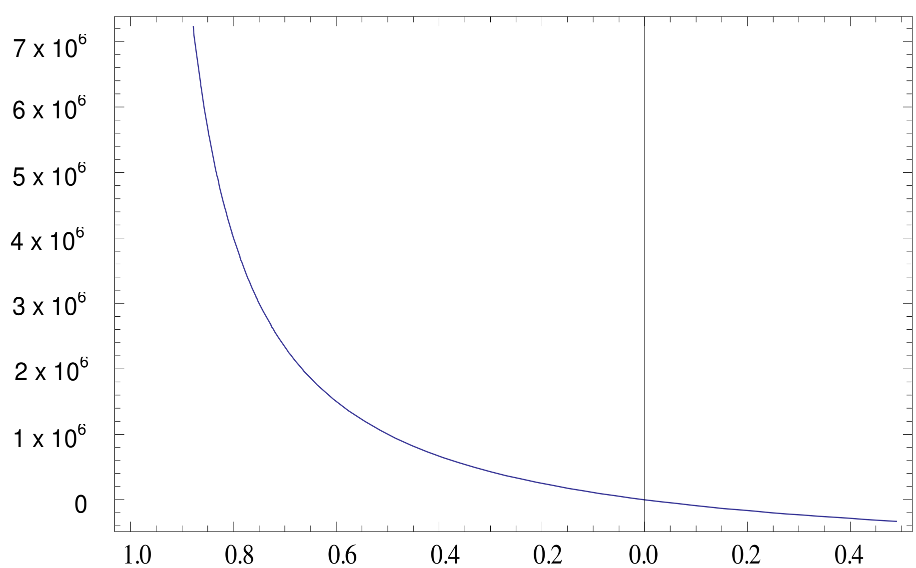

Figure 4. Comparison of stress component τ_{yy} in auxetic and non-auxetic material

$$\tau_{xx\infty} = \tau_{zz\infty} = -\frac{q\nu}{(1-\nu)}(1+\varepsilon^{*}) \tag{31}$$

where, the coupling existing between elastic and temperature field is given by the parameter

$$\varepsilon^{*} = \frac{(1-2\nu)}{(1+\varepsilon)\nu}$$

RESULTS AND DISCUSSION

The object of our interest is a layered plate subjected to a sudden uniform normal pressure of intensity q applied at the faces $x = \pm\frac{h}{2}$ with temperature $T = 30\text{o}K$. To study the auxetic and non-auxetic behaviour, the material chosen for the plate is Alumina/Silicon. The numerical data necessary to do the calculations are collected in Table 1.

The auxetic and non-auxetic behavior of the material due to taking values of Poisson's ratio $-1 < \nu < 0.5$ is studied. The graphs are drawn to show the variation in temperature and stress distribution due to external pressure. Variations in temperature field for the negative

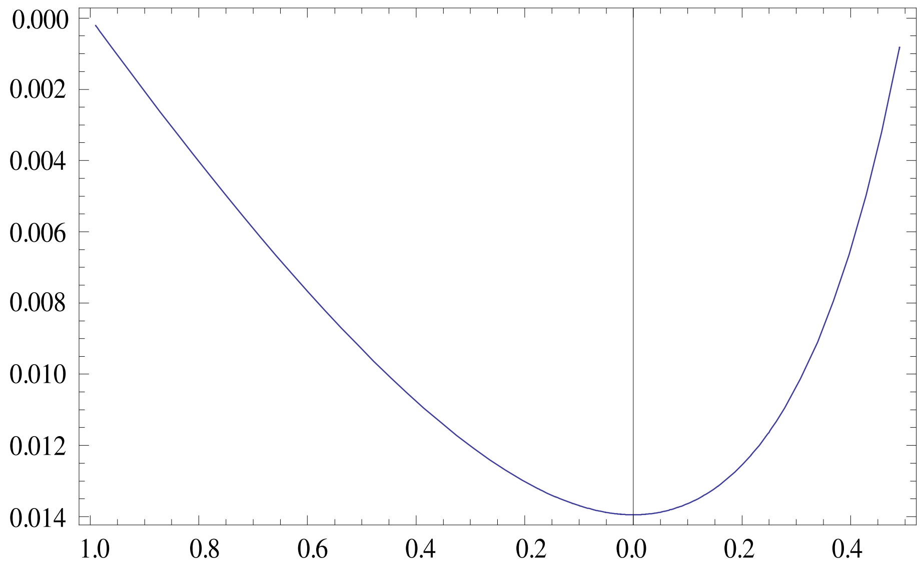

Figure 5. Horizontal displacement in auxetic and non-auxetic material.

and positive values of Poisson's ratio are shown in Figure 2 and the variation in normal stresses τ_{xx} and τ_{yy} are shown in Figures 3 and 4 respectively.

Additionally, the results for displacement field $\frac{\partial u}{\partial x}$, are shown in Figure 5. These results show clear difference in response of auxetic material versus non-auxetic material to externally applied load in a thermal elastic plate. It indicates that a further study of this problem, especially the one which involves layers or composites with non – auxetic/auxetic materials should be pursued.

APPENDIX I

(a) We define non-dimensional quantities as the following:

$$x' = \frac{x}{l}, \qquad u_j' = \frac{u_j}{l}, \qquad \Theta_j' = \frac{\Theta_j}{T_0}, \qquad x' = \frac{x}{l}$$

(b) Limit theorem

$$f = (\infty) = \lim_{s \to 0} s\,\bar{f}(s)$$

NOMENCLATURE

$$\lambda_j = \frac{E_j \nu_j}{(1+\nu_j)(1-2\nu_j)}, \quad \mu_j = \frac{E_j}{2(1+\nu_j)}$$

Lamé's constants

ρ_j : Density

α_j : Coefficient of thermal expansion;

K_j : Thermal conductivity;

C_j : Specific heat of material under constant volume

$\theta_{,j}$: Temperature gradient

h : Heat transfer coefficient

b : Body forces

κ : $\frac{K_j}{\rho_j C_j}$

ν : Poisson's ratio

q : Pressure

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